Technische Universität München
Zentrum Mathematik

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Sheet 9

Prof. Dr. Claudia Scheimbauer
Anja Švraka

# Bordisms and Topological Field Theories [MA5133]

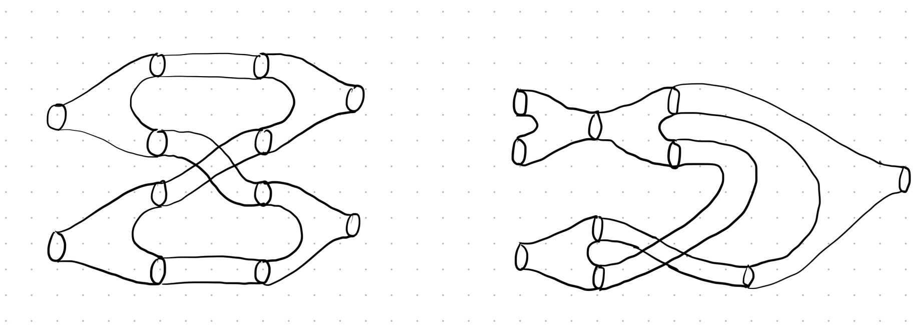
**Exercise 1.** ***Example of a R-matrix***
Let $V$ be a 2 dimensional vector space. Let $R \in \mathrm{End}(V \otimes V)$ be given by

$$R(q,\lambda) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \lambda & 1-q\lambda & 0 \\ 0 & 1-q^{-1}\lambda & \lambda & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

where $q$ is a non-zero scalar and $\lambda$ is an arbitrary scalar. Show that $R(q,\lambda)$ satisfies the Yang-Baxter equation if

$$\lambda + \lambda^3 = (q+q^{-1})\lambda^2.$$

**Exercise 2.** ***Bringing 2 dimensional cobordisms to normal form***
Use the relations of a (co)commutative Frobenius algebra step by step to bring the cobordisms below to normal form:



**Exercise 3.** ***Braid category is the coproduct of delooping groupoid of braid groups***
**Definition 1.** For a group $G$, there is a category with a single object, where morphisms are elements of the group $G$, and the group operation gives the composition. Such a category is a groupoid, usually denoted again by $G$, and is called **delooping groupoid**.

Show that the category of braids is equivalent to the coproduct of the delooping groupoids of the braid groups

$$\mathrm{Braid} \simeq \coprod_k B_k.$$