Winter semester 2023/24 Claudia Scheimbauer

Bordisms and Topological Field Theories

1. Topic of course

Studying manifolds up to diffeomorphism is very difficult. However, if we instead study manifolds up to "cobordism" and consider the disjoint union we obtain very computable groups. In fact, product of manifolds gives a cobordism ring. In the 1980's, Atiyah and Segal realized that the notion of cobordism naturally appears when decribing topological field theories mathematically. In the course we will encounter these notions.

2. Outline

Our program will roughly be:

- Introduction to the ideas behind (co)bordism and topological field theories
- Cobordism group Ω_n. First examples: 1d oriented and unoriented cobordisms, $\Omega_0^{(or)}$, and Ω_1.
- *Insert:* Smooth manifolds and tangential structures: orientations, possibly spin structures
- Two-dimensional case: recollection of classification of 2-dimensional manifolds. Consequences for Ω_2^{or}, outlook on Ω_2 – why algebraic topology quickly becomes necessary
- Cobordism ring
- *Insert:* Symmetric monoidal categories
- Definition of topological field theories (TFTs) after Atiyah and Segal [**?**]: bordism categories and tensor products, first consequences
- 1d classification and duality
- 2d classification and commutative Frobenius algebras – this will take some time! [**?**]
- Towards 3-dimensional Chern-Simons/Witten-Reshetikin-Turaev theory – we will follow [**?**], roughly Chapters 1-5. This needs:
  - Braid groups and their representations
  - Braided monoidal categories which we will have already introduced
  - Hopf algebras
- *Option:* more advanced cobordism theory stuff. generalized (co)homomology theory, Computation, characteristic classes [**?**]

3. Grade bonus option: outlook talks

If everyone agrees and there are not too many participants, we could try out the following: you could earn a grade bonus by giving a talk about a topic which goes beyond the main content of the course. Topics could include

- Jones polynomial as a knot invariant [**?**]
- Topological quantum field theories from a physics perspective (for a physicist)
- More cobordism theory (for a mathematician with some algebraic topology background)
- From a tangle invariant to a TFT: surgery of manifolds.
- generalized (co)homomology theories and (co)bordism
- Towards classifications in higher dimensions (For an advanced student)